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# Non-Perturbative Fine-Tuning in Approximately Supersymmetric Models

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## Abstract

We present two Fermi-Bose models with an approximate supersymmetry and which can be solved numerically with great accuracy using the renormalization group method. The bosonic parts of these models consist in Dyson's hierarchical model with one and two scalar components respectively. We discuss the question of the perturbative cancellations of divergences and compare with the non-perturbative fine-tunings necessary to keep the renormalized scalar mass small in cut-off units. We show evidence for non-perturbative cancellations of quantum corrections, however, we were not able to achieve exact cancellations without fine-tuning.

The fact that the bare parameters of a scalar field theory require a fine-tuning in order to keep the renormalized mass small in cut-off units is usually regarded as an argument against fundamental scalars [1]. A possible resolution of this inelegant feature consists in adding degrees of freedom in such a way that the quantum fluctuations cancel, making small scalar masses a more natural outcome.

From a non-perturbative renormalization group [2] analysis, a relevant direction is necessary in order to describe massive particles, making the fine-tuning process unavoidable. This situation can be seen very simply in the case of a free scalar theory with a cut-off. In this example, the fact that the bare mass ( $m_B$ ) has to be small (in cut-off units) in order to get a small physical mass ( $m_R$ ) is obvious since these quantities are identical. On the other hand in an interacting scalar theory, we usually need to take  $m_B^2$  negative and large in absolute value and also to adjust many digits of this quantity in order to get a small  $m_R$ . The difference between the two situations is that in the second case, there is no bare quantity which controls the size of  $m_R$ . One would like to understand under which circumstances the inclusion of fermions allows us to obtain a small  $m_R$  whenever we choose a small  $m_B^2$ .

There are known four-dimensional examples [3] where one can cancel the perturbative quadratic divergences by imposing simple relations between the Yukawa couplings and the scalar quartic couplings. However, it is not clear that there exists a non-perturbative regularization which fully preserves the perturbative naturalness.

In the following, we present two models where there is an equal number of fermions and bosons and which can be solved non-perturbatively with great accuracy [5]. The bosonic part of these model is a Dyson's hierarchical model [4]. In this model, the renormalization group transformation maps the local measure into another local measure. The price to pay for this simplifying feature is that the kinetic term is not ultra-local. The free action for  $N$  massless scalar fields  $\phi_x^{(i)}$  reads

$$S_B^{free} = \frac{1}{2} \sum_{x,y,i} \phi_x^{(i)} D_{xy}^2 \phi_y^{(i)} , \quad (1)$$

where  $x$  and  $y$  run over the sites and  $i$  from 1 to  $N$ . The explicit form of  $D_{xy}^2$  is given below in Eq. (6). The action for free massless fermions reads

$$S_F^{free} = \sum_{x,y,i} \bar{\psi}_x^{(i)} D_{xy} \psi_y^{(i)} , \quad (2)$$

where the  $\psi_x^{(i)}$  and  $\bar{\psi}_x^{(i)}$  are Grassmann numbers integrated with a measure

$$\int \prod_{x,i} d\psi_x^{(i)} d\bar{\psi}_x^{(i)} . \quad (3)$$

As indicated by the notation, we have

$$D_{xy}^2 = \sum_z D_{xz} D_{zy} . \quad (4)$$

The free action  $S_B^{free} + S_F^{free}$  is invariant at first order under the transformation

$$\begin{aligned}
\delta\phi_x^{(i)} &= \epsilon\bar{\psi}_x^{(i)} + \psi_x^{(i)}\bar{\epsilon} \\
\delta\psi_x^{(i)} &= \epsilon\sum_x D_{xy}\phi_y^{(i)} \\
\delta\bar{\psi}_x^{(i)} &= \bar{\epsilon}\sum_x D_{xy}\phi_y^{(i)} .
\end{aligned} \tag{5}$$

The  $\epsilon$  and  $\bar{\epsilon}$  are Grassmann numbers. Integration by part or Leibnitz's rule cannot be used for  $D_{xy}$  and the order  $\epsilon\bar{\epsilon}$  variations do not cancel.

We now give the explicit form of  $D_{xy}^2$  at finite volume. For a hierarchical model with  $2^{n_{max}}$  sites, we label the sites with  $n_{max}$  indices  $x_{n_{max}}, \dots, x_1$ , each index being 0 or 1. In order to understand this notation, one can divide the  $2^{n_{max}}$  sites into two blocks, each containing  $2^{n_{max}-1}$  sites. If  $x_{n_{max}} = 0$ , the site is in the first box, if  $x_{n_{max}} = 1$ , the site is in the second box. Repeating this procedure  $n$  times (for the two boxes, their respective two sub-boxes, etc...), we obtain an unambiguous labeling for each of the sites. With these notations,

$$S_B^{free} = -\beta_B \sum_{n=1}^{n_{max}} \left(\frac{c_B}{4}\right)^n \sum_{x_{n_{max}}, \dots, x_{n+1}, i} \left( \sum_{x_n, \dots, x_1} \phi_{(x_{n_{max}}, \dots, x_1)}^{(i)} \right)^2 + \frac{\beta_B c_B}{2 - c_B} \sum_{x_{n_{max}}, \dots, x_{n+1}, i} \left( \phi_{(x_{n_{max}}, \dots, x_1)}^{(i)} \right)^2 . \tag{6}$$

The index  $n$  corresponds to the interaction of the total field in blocks of size  $2^n$ . The constant  $c_B = 2^{1-2/D}$  is a free parameter which controls the decay of the iterations with the size of the boxes and can be adjusted in order to mimic a  $D$ -dimensional model. Similarly the free massless fermionic action reads

$$S_F^{free} = -\beta_F \sum_{n=1}^{n_{max}} \left(\frac{c_F}{4}\right)^n \sum_{x_{n_{max}}, \dots, x_{n+1}, i} \left( \sum_{x_n, \dots, x_1} \bar{\psi}_{(x_{n_{max}}, \dots, x_1)}^{(i)} \right) \left( \sum_{x_n, \dots, x_1} \psi_{(x_{n_{max}}, \dots, x_1)}^{(i)} \right) \tag{7}$$

$$+ \frac{\beta_F c_F}{2 - c_F} \sum_{x_{n_{max}}, \dots, x_{n+1}, i} \bar{\psi}_{(x_{n_{max}}, \dots, x_1)}^{(i)} \psi_{(x_{n_{max}}, \dots, x_1)}^{(i)} , \tag{8}$$

with  $c_F = 2^{1-1/D}$ . Using the techniques explained in [6], one can show that the fermionic operator is the square root of the bosonic operator (see Eq. (4)) provided that

$$\frac{\beta_F c_F}{2 - c_F} = \left( \frac{\beta_B c_B}{2 - c_B} \right)^{\frac{1}{2}} \tag{9}$$

We now introduce local interactions. The Grassmann nature of the fermionic fields restricts severely the type of interactions allowed. For instance, for one flavor ( $N = 1$ ), the most general local measure is

$$\mathcal{W}(\phi, \psi, \bar{\psi}) = W(\phi) + \psi \bar{\psi} A(\phi) \tag{10}$$

For convenience, we will always reabsorb the second term of Eqs. (6) and (8) which are local, in the local measure. In the following calculations,  $W(\phi)$  will take the Landau-Ginzburg (LG) form:

$$W(\phi) \propto e^{-((\frac{\beta_B c_B}{2 - c_B}) + \frac{1}{2} m_B^2 \phi^2 + \lambda_B \phi^4)} . \tag{11}$$

If the two functions  $W$  and  $A$  are proportional, the fermionic degrees of freedom decouple. The renormalization group transformation takes the form

$$W \rightarrow 2A \star W \quad (12)$$

$$A \rightarrow 2\beta_F A \star W + \left(\frac{4}{c_F}\right) W \star W \quad (13)$$

where the  $\star$  operation means a convolution, a multiplication by an exponential and a rescaling of the new field. More precisely

$$A \star B(\phi) \equiv e^{\frac{\beta_B}{2}(\phi^2)} \int d\phi' A\left(\frac{(\phi 2c_B^{-\frac{1}{2}} - \phi')}{2}\right) B\left(\frac{(\phi 2c_B^{-\frac{1}{2}} + \phi')}{2}\right), \quad (14)$$

The introduction of a Yukawa coupling can be achieved by allowing a linear term in  $A(\phi)$ . Such a term breaks explicitly the  $Z_2$  symmetry of the LG measure. Such a model is characterized by a sudden change from the symmetric phase behavior to the broken phase behavior followed by unexpectedly long low-temperature “shoulders” (see [7] for an explanation of this terminology).

A richer behavior is observed in the case of the two flavors ( $i = 1, 2$ ) models. In the following we have restricted our investigation to the type of bilinear coupling appearing in the Wess-Zumino [3] model, namely

$$\begin{aligned} \mathcal{W}(\phi^{(i)}, \psi^{(i)}, \bar{\psi}^{(i)}) = & W(\phi^{(i)}) + A(\phi^{(i)})(\bar{\psi}^{(1)}\psi^{(1)} + \bar{\psi}^{(2)}\psi^{(2)}) + \\ & B(\phi^{(i)})\psi^{(1)}\psi^{(2)} - B^*(\phi^{(i)})\bar{\psi}^{(1)}\bar{\psi}^{(2)} + T(\phi^{(i)})\bar{\psi}^{(1)}\psi^{(1)}\bar{\psi}^{(2)}\psi^{(2)}. \end{aligned} \quad (15)$$

For convenience, we again absorb the local parts of Eqs. (6) and (8). This is not the most general measure, however it closes under the renormalization group transformation which takes the form

$$\begin{aligned} W & \rightarrow (W \star T + A \star A + B \star B^*) \equiv W' \\ A & \rightarrow \beta_F W' + \frac{4}{c_F} A \star T \\ B & \rightarrow \frac{4}{c_F} B \star T \\ T & \rightarrow \frac{8}{c_F^2} T \star T + \beta_F \frac{8}{c_F} A \star T + (\beta_F)^2 W'. \end{aligned} \quad (16)$$

In addition we will impose that the function  $B$  have the following form:

$$B(\phi^{(i)}) = (\phi^{(1)} + i\phi^{(2)})P((\phi^{(1)})^2 + (\phi^{(2)})^2), \quad (17)$$

while  $W$ ,  $A$  and  $T$  are  $O(2)$ -invariant. The model is then invariant under the R-symmetry

$$\begin{aligned} (\phi^{(1)} + i\phi^{(2)}) & \rightarrow e^{i\theta}(\phi^{(1)} + i\phi^{(2)}) \\ \psi^{(j)} & \rightarrow e^{-i\frac{\theta}{2}}\psi^{(j)} \\ \bar{\psi}^{(j)} & \rightarrow e^{i\frac{\theta}{2}}\bar{\psi}^{(j)}. \end{aligned} \quad (18)$$

We now present three numerical calculations performed with the second model. In all cases we will set  $D = 4$  in  $c_B$  and  $c_F$ . In the following, we have chosen the value of  $\beta_B$  and  $\beta_F$  in such a way that

$$\frac{\beta_F c_F}{2 - c_F} = \left( \frac{\beta_B c_B}{2 - c_B} \right)^{\frac{1}{2}} = 1, \quad (19)$$

in order to make the perturbative expansion more similar to usual Feynman diagram's calculations.

First, we consider the case where the fermions decouple from the bosons.  $W$  takes a LG form

$$W(\phi) \propto e^{-\left(\left(\frac{\beta_B c_B}{2 - c_B}\right) + \frac{1}{2} m_B^2\right) \sum_i (\phi^{(i)})^2 + \lambda_B (\sum_i (\phi^{(i)})^2)^2}. \quad (20)$$

The value of  $m_R^2$ , defined as the inverse of the zero-momentum two-point function, is shown in Fig. 1. as a function of  $m_B^2$ . These quantities are expressed in cut-off units. For reference we have also displayed the one-loop perturbative result and the trivial gaussian result. One sees that the scalar self-interaction moves  $m_R^2$  up and  $m_R^2 \simeq 0.2$  when  $m_B^2$  goes to zero. The one-loop result is quite good when  $m_R^2$  is large enough but deteriorates when this quantity becomes smaller.

In the second calculation, we consider a bosonic model with a bare mass  $m_B$  and  $\lambda_B = 0$  coupled to a fermion with the following couplings:

$$\begin{aligned} A &= (-1 - m_B)W \\ P &= g_y W \\ T &= ((-1 - m_B)^2 + g_y^2 ((\phi^{(1)})^2 + (\phi^{(2)})^2))W. \end{aligned} \quad (21)$$

The results are shown in Fig. 2 for  $g_y = \sqrt{0.08} \simeq 0.28$ . One sees that the Yukawa coupling moves  $m_R^2$  down. For  $m_B^2 \simeq 0.094$ ,  $m_R$  becomes 0 and for smaller of  $m_B^2$ , we enter the broken symmetry phase.

We have then repeated the second calculation with  $\lambda_B = 0.01$  instead of 0. In perturbation theory, the one-loop quadratic divergence cancel when  $m_B = 0$  and

$$8\lambda_B = g_y^2, \quad (22)$$

which justifies our choice of coupling constant. The results are shown in Fig. 3. One sees that the Yukawa coupling in part cancels the effects of the scalar self-interaction, however, the cancellation is not as good as in the one-loop formula where  $m_R$  goes to zero when  $m_B^2$  goes to zero. Instead, we found numerically that  $m_R^2 \simeq 0.044$  when  $m_B^2$  goes to zero. A summary of the three numerical results is shown in Fig. 4.

It is possible to fine-tune  $g_y$  in order to get  $m_R = 0$ . An example is shown in Fig. 5 for  $\lambda_B = 0.01$  and  $m_B^2 = 0.01$ . We see that there exist a critical value of  $g_y$  which is approximately 0.46 and where  $m_R$  becomes 0. For larger values of  $g_y$ , we enter the symmetry broken phase. An essentially similar figure is obtained for  $m_B^2 = 0$ . In both cases, the exact critical value of  $g_y$  is about 50 percent larger than the perturbative one.

In conclusion, we have shown that the idea of canceling the quantum correction inspired by perturbation theory have qualitatively a non-perturbative counterpart. However, we have not found a way to make this cancellation very accurate or exact without fine-tuning.

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# FIGURES

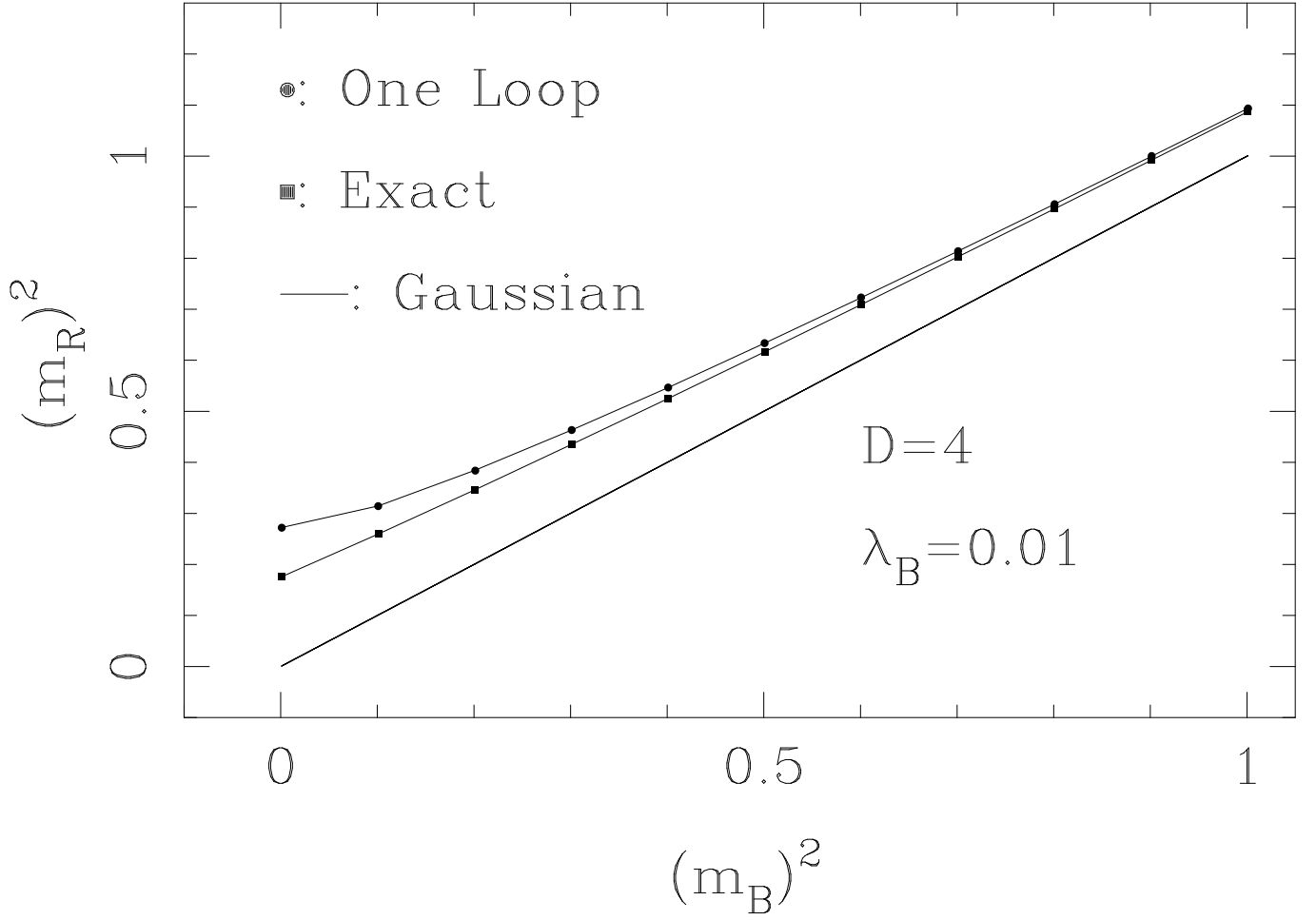


FIG. 1. The renormalized mass as a function of the bare mass in a bosonic  $O(2)$  model with bare quartic coupling fixed to 0.01.



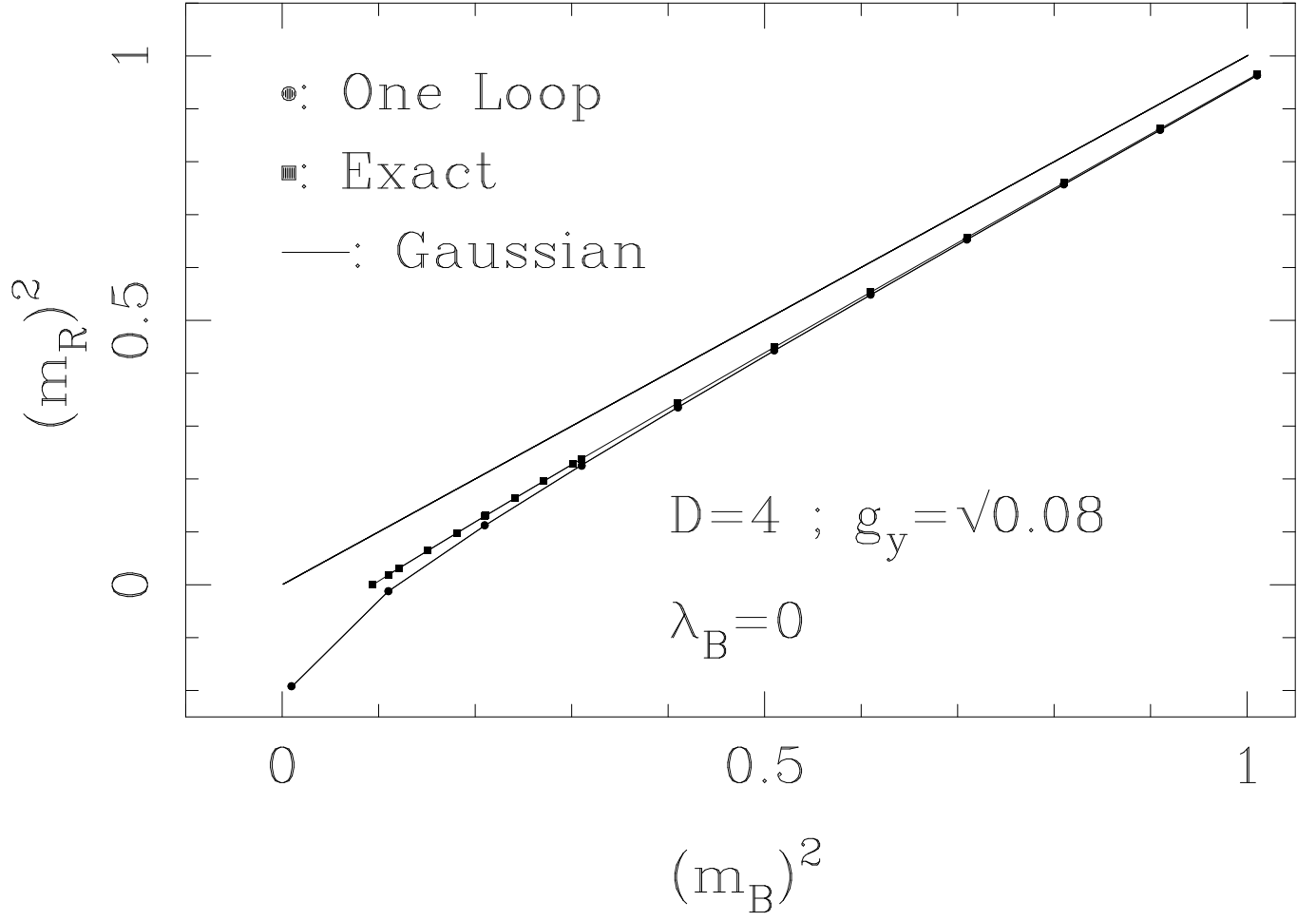


FIG. 2. The renormalized mass as a function of the bare mass in a bosonic  $O(2)$  model with bare quartic coupling fixed to 0 and a Yukawa coupling equal to  $\sqrt{0.08}$

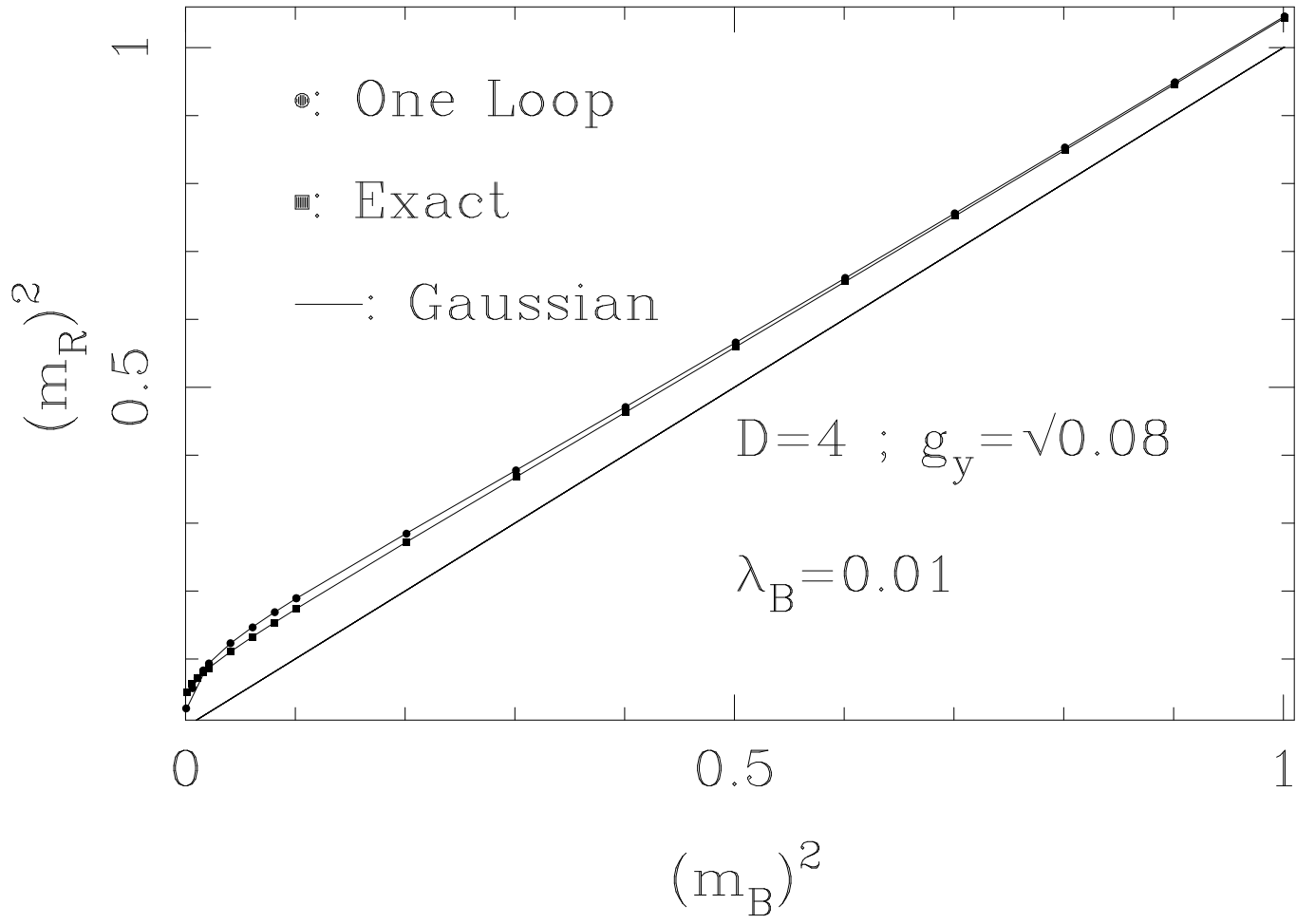


FIG. 3. The renormalized mass as a function of the bare mass in a bosonic  $O(2)$  model with bare quartic coupling fixed to 0.01 and a Yukawa coupling equal to  $\sqrt{0.08}$ .

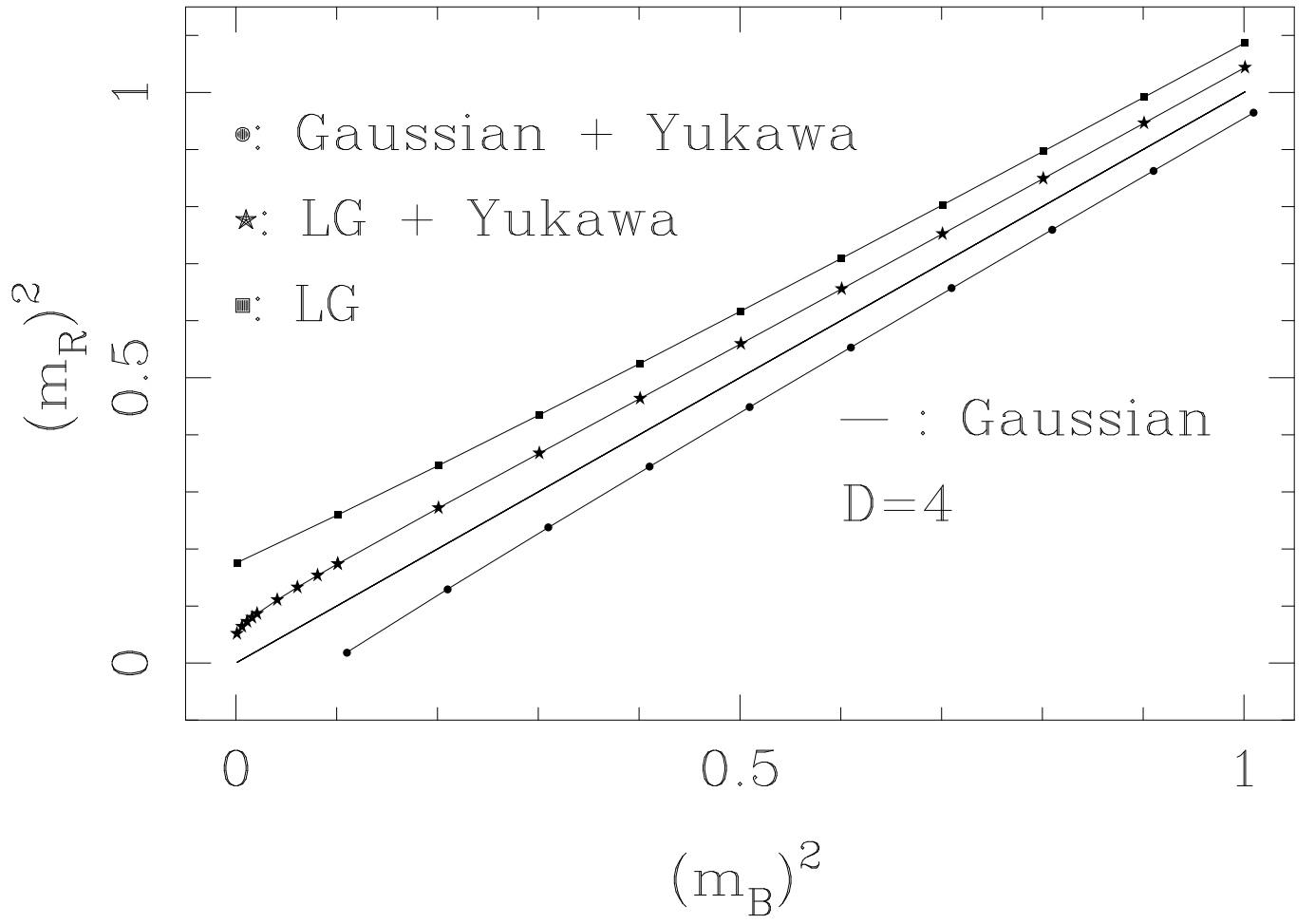


FIG. 4. The numerical results of Figs. 1, 2 and 3 combined together.

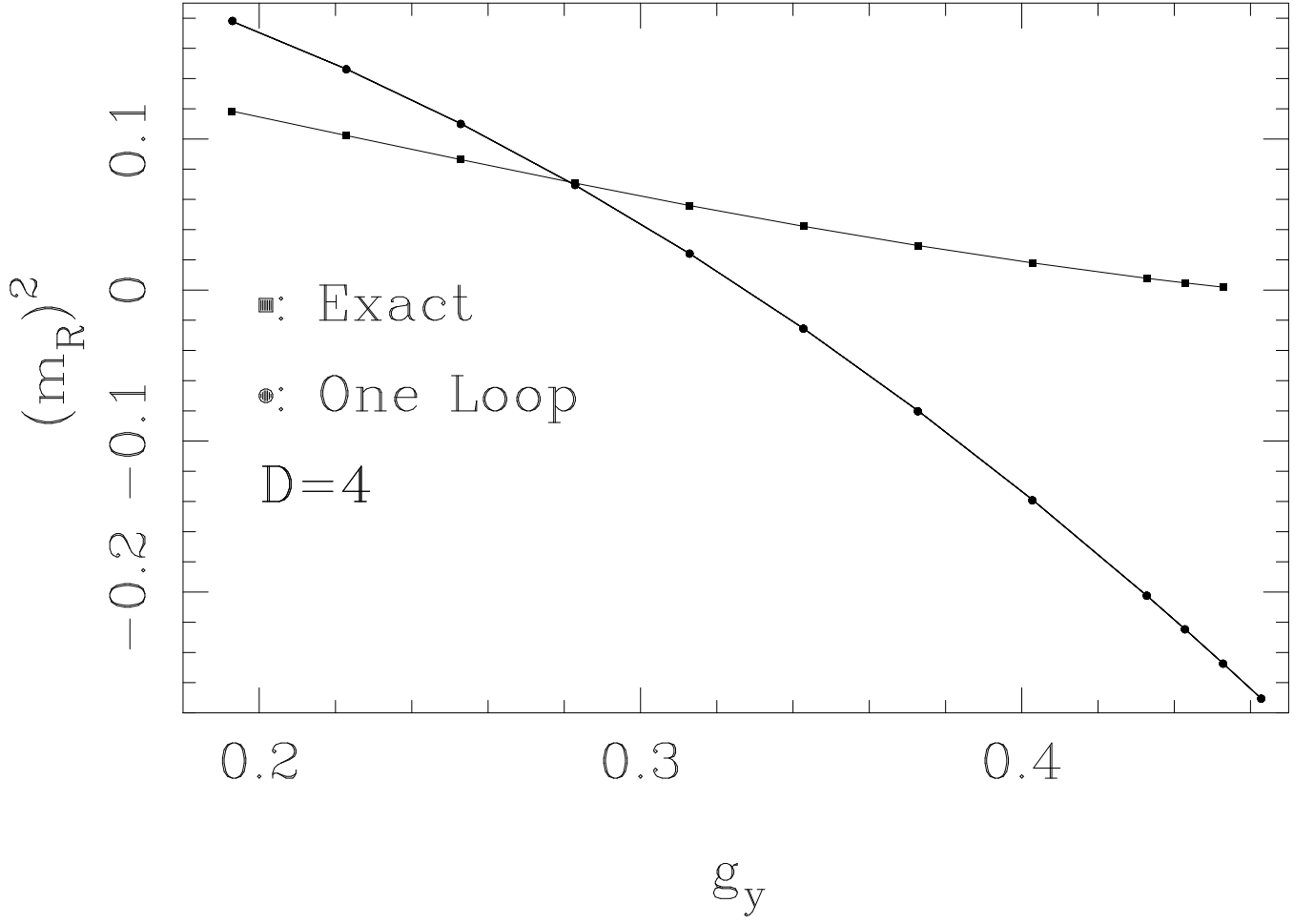


FIG. 5. The renormalized mass as a function of the Yukawa coupling in a bosonic  $O(2)$  model with bare quartic coupling fixed to 0.01 and a bare mass  $m_B^2 = 0.01$ .